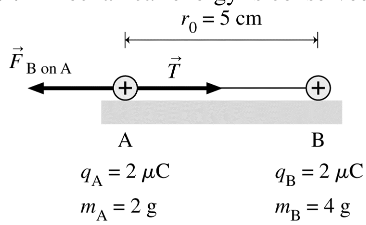
29.41. Model: Mechanical energy is conserved. Metal spheres are point particles and they have point charges.

Visualize:



Solve: **(a)** The system could have both kinetic and potential energy, although here $K = 0$ J. The energy of the system is

$$E_0 = K_0 + U_0 = 0\text{ J} + \frac{q_A q_B}{4\pi\varepsilon_0 r_0} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{0.05\text{ m}} = 0.720\text{ J}$$

(b) In static equilibrium, the net force on sphere A is zero. Thus

$$T = F_{\text{B on A}} = \frac{|q_A||q_B|}{4\pi\varepsilon_0 r_0^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.05\text{ m})^2} = 14.4\text{ N}$$

(c) The spheres move apart due to the repulsive electric force between them. The surface is frictionless, so they continue to slide without stopping. When they are very far apart ($r_1 \to \infty$), their potential energy $U_1 \to 0$ J. Energy is conserved, so we have

$$E_1 = K_1 + U_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 + 0\text{ J} = E_0$$

Momentum is also conserved: $P_{\text{after}} = m_A v_{A1} + m_B v_{B1} = P_{\text{before}} = 0$ kg m/s. Note that these are *velocities* and that v_{A1} is a negative number. From the momentum equation,

$$v_{A1} = -\frac{m_B v_{B1}}{m_A}$$

Substituting this into the energy equation,

$$E_0 = \tfrac{1}{2}m_A\left(-\frac{m_B v_{B1}}{m_A}\right)^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}m_B\left(\frac{m_B}{m_A}+1\right)v_{B1}^2$$

$$\Rightarrow v_{B1} = \sqrt{\frac{2E_0}{m_B(m_B/m_A+1)}} = \sqrt{\frac{2(0.720\text{ J})}{(0.004\text{ kg})(4\text{ g}/2\text{ g}+1)}} = 10.95\text{ m/s}$$

Using this result, we can then find $v_{A1} = -m_B v_{B1}/m_A = -21.9$ m/s. These are the velocities, so the final *speeds* are 21.9 m/s for the 2 g sphere and 10.95 m/s for the 4 g sphere.